 Adam Narkiewicz, anarkiewicz@wp.pl

A new market model for a fishery

Abstract (130 words)

We propose a new intra-seasonal model for a fishery that allows for explaining both efficiency gains and welfare effects related to the introduction of individual quotas in fisheries previously operating as regulated open access. Our model is also suitable for analyzing international fisheries where one country decides to adopt new regulation meanwhile the other does not. We derive comparative statics which allows us to form some policy recommendations. We also calibrate our model so that it resembles the situation in North Pacific Halibut Fishery and test it against some empirical evidence as well as compare its results to results found in other research. We conclude that our model better describes reality than models used in previous research. We also simulate counterfactual outcomes which allows us to formulate further policy recommendations.

Keywords: fishery, international fishery, market model, individual quotas, derby fishing

1. Introduction

Fisheries entail many economic problems. Probably first to notice by general public, and still pervasive was the problem of overfishing. Unregulated behavior of fishermen can and often do lead to the collapse of the stock (see Clark, 1976, p. 32). This problem arises as an effect of so called stock externality and, at least in theory, can be effectively solved by imposing total annual catches that is seasonal quotas determined by biologists usually working with the fishery regulatory agency (Clark, 1980). Once this measure is effectively implemented, another economic problems draw attention. First in line is a problem of derby fishing (Clark, 2006, p. 77) whereas fishermen tend to increase their fishing effort to exorbitant levels as they want to catch as much fish as possible before the annual quota is met. This is a simple game theory exercise to show that derby fishing is an equilibrium behavior.

Initially, the inefficiency and welfare losses under derby fishing were attributed to the fact that very high levels of effort are not economically justified and result in rent dissipation even though a substantial profit could be made given the amount of fish to be caught (Weninger, 1998). The solution to this problem is to introduce individual quotas and was originally suggested by Christy (1973). This regulatory regime comes in many flavors, but the essence is that each fisherman (or a fishing vessel) has a permit to catch specific amount of fish. This reduces incentives for derby fishing, since nobody can increase their catch over the limit. As numerous studies show, this solution indeed increases profits from fishing, which can be proxied by price of individual quotas, as long as their trading or leasing is possible (see for example Newell et al. 2005).

A generally unexpected consequence of introduction of individual fishing quotas is that they lead to an increase in price, usually attributed to an increase in quality (see for example Herrmann, 1996). Fishermen have more time to handle the fish and they don't glut the market with the annual catch over a short period of time. Thus, much more fish can be marketed as fresh rather than frozen which also contributes to perceived increase in quality.

Various studies tried to estimate effects of individual quotas and build models that would allow for useful predictions, for instance what would be an effect of increasing the season length on ex vessel price/fishermen revenue (as an example see Herrmann and Criddle, 2006)? These models are usually empirical and focused on making predictions with respect to a single variable. There is a noticeable lack of a theoretical model that would entail both production efficiency gains and effects of price increase and allow for welfare analysis. Such a model, would help to explain and understand benefits from transition from derby fishing to individual quotas and potentially improve further empirical practice. The goal of this paper is to fill this gap and propose such a model.

There are a considerable number of papers trying to model fishery in the inter-seasonal setting. The whole literature was launched by Gordon (1954) and further developed, most notably with milestone papers by Copes (1970) and Clark (1980). Nevertheless, all these papers are focused on dealing with stock externality rather than modeling welfare gains. In order to proceed with profit and consumer surplus analysis, it is necessary to construct an intra-seasonal model based on assumption that problem of overfishing has been already efficiently solved. One of such models was proposed by Homans and Wilen (2005). This is probably the closely related paper to the topic considered herein, the authors however focus on the demand side and it is not possible with their model to draw many conclusions with respect to input stuffing. Moreover, despite focusing on demand side authors abstain from conclusions about welfare.

The paper by Homans and Wilen provides some interesting insights but we consider their model to have too many glitches to be further extended and adapted for welfare analysis and modeling of input stuffing. Although we agree with some of its assumptions, there are many assumptions that are unnecessary and others that are counterintuitive. To the first category belong inclusion of discounting, even though we are operating in a single period (that is one year), and inclusion of stock effects which were originally developed to facilitate inter-temporal analysis rather than to model single period equilibrium (see Clark, 2006). Stock effects in their model result in a price increasing over the season and thereafter as well as in decreasing catch. Because authors assume constant effort throughout the season this results in different profit margins for fishermen and thus leads to negligence of the fact that fishermen are profit optimizing. It seems counterintuitive that harvest rate is assumed to be proportional to instantaneous effort – congestion and hurry should contribute to decreasing returns to effort. Authors assume that capital is specialized to a single fishery with no alternative uses which is clearly untrue in the light of other research (for example Casey et al. 1995). Homans and Wilen also assume that frozen fish is not consumed whenever fresh fish is, which neglects distant sources of demand, where product can be delivered only as frozen. Finally, their model does not allow for any comparative statics. All this shortcomings justify creating an entirely new model.

The paper proceeds as follows. The second chapter describes general ideas behind the model and proposes systems of equations characterizing market equilibria in four distinct situations: 1) when a fishery is regulated with individuals quotas, 2) when the fishery is governed by regulated open access with derby fishing, 3) when fishery is split between two countries with comparable shares in the fishery, 4) when fishery is split between two countries of whom one is dominating. Third chapter describes general properties of market equilibria in this model, including existence and uniqueness results as well as comparative static. Finally, fourth chapter contains numerical simulations which are based on North Pacific Halibut Fishery. The results are compared to results obtained in other papers. The paper ends with conclusions.

2. Model

The model is based on two main components: instantaneous welfare function $W(q_H, q_L)$ and instantaneous production function $q(s, E)$. Instantaneous welfare function (from now on just welfare function) represents aggregate utility derived by entire buyer population from consuming q_H units of high quality fish and q_L units of low quality fish within a unit period of time. Since this is a partial equilibrium modal, complete aggregate utility encompassing also other goods is quasi-linear and W is expressed in monetary units (US dollars). This simplifying assumption should closely approximate reality as long as consumption of fish constitutes a small fraction of buyers' budgets and income effects of changes in fish prices are negligible. W satisfies standard properties of welfare functions, that is it is increasing in both goods and strictly concave. It is also twice continuously differentiable, the two goods are substitutes and cross price effects are weaker than own price effects.

Instantaneous production function q informs how much fish was captured in a unit of time given the effort undertaken by fleet E and s, the fraction of the entire fishery the fleet is operating in. Effort is expressed in monetary units (US dollars) and can be understood as a sum of variable costs (crew remuneration, fuel costs, etc.) and capital opportunity costs expanded by the fleet in a unit of time. We assume that the fishery is homogeneous, so expanding both effort and size of the area the fleet operates in by the same fraction should increase the yield by the same fraction. Therefore, q is homogeneous of degree 1, that is $q(ts, tE) = tq(s, E)$. For simplicity, q is also the same no matter what part of fishery the effort is undertaken in. Moreover, $q(s, 0) = 0$, q is increasing in both arguments, and due to congestion and rising opportunity costs $\frac{\partial^2 q}{\partial E^2} < 0$. Note that, as often happens in a fishery, due to stock depletion it may take more effort to obtain the same yield, so q changes over time. However, in this model, q will be kept constant as explaining effects of variation in stock is not an objective of this paper.

Consumers are maximizing their utility, therefore, for given prices p_H and p_L, the demands can be found as solutions to the following maximization problem: $\max_{q_H \geq 0, q_L \geq 0} [W(q_H, q_L) - p_H q_H -$

$p_L q_L]$. Let's denote them as $D_H(p_H, p_L)$ and $D_L(p_H, p_L)$ for high- and low-quality fish respectively. Let $D_L(+\infty, p_L)$ denote the demand for low quality fish in absence of high quality fish, that is a solution to the problem: $\max_{q_L \geq 0}[W(0, q_L) - p_L q_L]$. It is convenient to let year be a unit of time. Then, a fishing season can be described by variable $0 \leq T \leq 1$, and consuming a q_H of high- and q_L of low-quality fish within this season will yield a consumer surplus of $T[W(q_H, q_L) - p_H q_H - p_L q_L] = TV(p_H, p_L)$, where prices are assumed to be constant over the entire period. $V(p_H, p_L) = \max_{q_H \geq 0, q_L \geq 0}[W(q_H, q_L) - p_H q_H - p_L q_L]$ is the indirect welfare function. q_H and q_L are consumption rates, or in other words consumption levels in a unit of time. Therefore in above example, the consumption of high- and low quality fish in the period of length T will be Tq_H and Tq_L respectively.

Similarly, fishermen are also maximizing their profits. They generally tend to equalize profit margins within fishing season by shifting effort to more profitable moments of time. They also tend to increase effort as long as they can make profit without violating quota constraints. High quality fish and low quality fish are considered to be fresh and frozen fish respectively. Fishermen catch the fish at sea and then, the yield can be marketed either as fresh or as frozen. Since fishermen are profit maximizing, they will chose option that gives them higher price. Therefore, if at a particular point in time, both fresh and frozen fish is available, their ex vessel price must be the same. Fresh fish has to be consumed instantly after it was captured. Frozen fish can be stored and resold at any other time. Whoever holds inventories with frozen fish, they are selling it at a moment in time which gives them maximum profit that is when the price for frozen fish is the highest. This in turn equalizes the price of frozen fish. Thus, p_L is constant over the whole year.

The two features of the model, namely zero costs of keeping frozen fish and no processing sector may seem to be overly simplistic. As long as freezing costs are very low, they can be neglected but in reality this is not likely to be the case. Moreover, it often happens that processors affect the market situation in the fishery. For example, it is commonly known that under TAC processors are able to extract some rents from the fishery, a privilege they largely lose under ITQs (Matulich et al., 1996). We can give two interpretations to this assumption. Either there is no processing sector and there isn't any other intermediary sector, thus fish is sold directly to consumers, or the processing sector (and other intermediaries) is perfectly competitive and buys fish as long as its price is below its marginal valuation. This keeps our welfare calculations valid. Even if this interpretations fall short of reality, it is still reasonable to make them, since we are not interested in interaction of processing sector with the rest of the industry and such a simplification should still allow us to draw some general conclusions. It would probably be possible to include processing sector in our market model, but then what about other intermediaries, like restaurants, which constitute another link in the chain between producers and consumers? The reality always eludes being captured in a single model and we are strongly convinced that this model arrives at a useful compromise between simplicity and plausibility.

Individual quotas

Let's assume the entire fishery is operating in Individual Quotas regulatory system which is implemented in the same way for all parts of the fishery. There is an exogenous season length T_{IQ}, determined by the regulator. Fishermen can expand their effort only within the season and each fishermen is allotted their own quota which sum to the total allowable catch Q. As mentioned above, the price of low quality fish is constant within the whole year. High quality fish cannot be traded outside the season. Whenever high quality fish is traded, its price is the same as the price of low quality fish. Therefore, fishermen can get only single price for their product regardless of when they fish. Average profit margin per unit of effort, that is $\frac{p_{IQ}q(1,E)}{E} - 1$ and is equalized over the entire season which in turn means equalizing of effort since the price is constant and $\frac{q(1,E)}{E}$ is a strictly decreasing function of E.

Market equilibrium must satisfy two conditions. The markets must clear and the quota constraint must be met. These conditions are respectively:

1) $T_{IQ}\left(D_H(p_{IQ}, p_{IQ}) + D_L(p_{IQ}, p_{IQ})\right) + (1 - T_{IQ})D_L(+\infty, p_{IQ}) = Q$
2) $T_{IQ}q(1, E_{IQ}) = Q$

There are two endogenous variables in this system, the ex-vessel price p_{IQ} and the instantaneous effort E_{IQ}. As a matter of fact, the market equilibrium is characterized by a system of two non-linear equations with two unknowns.

Derby fishing

Derby fishing is characterized by the fact that season length is endogenous. In reality, the regulator usually sets season length in anticipation of fleet's behavior. Season can be then closed prematurely if the total allowable catch has been exceeded. It seems thus plausible to assume that market equilibrium fishing season length is fully endogenous.

As in case of IQs, there is a single price for both fresh and frozen fish. The rationale is exactly the same – fishermen deciding whether to market fish as low- or high-quality effectively equalize their prices, and given the prices are equal, the efforts are equalized within fishing season. The additional feature of this system is that as long as fishery is profitable, that is profit margin per unit of effort is greater than zero, new effort will enter fishery. It can be in a form of new boats, bigger crews, more intensive efforts etc. As a result, fishery yields zero profit. This adds a new equation to the system characterizing market equilibrium:

3) $T_{DF}\left(D_H(p_{DF}, p_{DF}) + D_L(p_{DF}, p_{DF})\right) + (1 - T_{DF})D_L(+\infty, p_{DF}) = Q$
4) $T_{DF}q(1, E_{DF}) = Q$
5) $p_{DF}q(1, E_{DF}) = E_{DF}$

where the equations are respectively 3) market clearing condition, 4) quota constraint, and 5) zero instantaneous profit condition. The endogenous variables are T_{DF}, p_{DF}, and E_{DF} so we have system of three non-linear equations with three unknowns.

Mixed situation

The most complicated case that is going to be analyzed in this paper is a mixed case, where one of the two countries participating in a fishery adopts individual quotas meanwhile the other country retains its derby fishing regulation. There are two cases in this scenario, depending on the relative size of the country adoption the new regulatory system. In both cases, it is going to be assumed that the fishing season in the derby country is entirely within the fishing season of the individual quota country. The calculations are easier to understand if we think that both seasons start when the year starts (at time 0), then season in the derby country ends at endogenously determined time T_{OLD} followed by ending of season in quota country at exogenously determined time T_{NEW}, where $0 < T_{OLD} \leq T_{NEW} \leq 1$. However, the calculations are still valid if neither season begins at time 0 (as long as we permit inter-temporal trade in fish), if the quota season is split into separate time intervals and if derby season has several separate openings.

Let's start with a case when the country first to adopt quota system is a big country. This case is characterized by the fact, that instantaneous production of this country is big enough to single-handedly satisfy the demand for high quality fish at the price level of low quality fish. In other words, inside quota country fishing season and outside derby country fishing season, quota country fishermen have to market their fish both as high- and low-quality. They obviously don't want to market their fish only as low quality, since the price of fresh fish would jump very high. If they market their fish only as high quality, the amount they produce is so high that the price they can get is below the price of low quality fish. Therefore they participate in both markets and the prices are equal. This implies that the price is the same for both types of fish and equal throughout the entire quota season and the rest of the year for low quality fish.

Fishermen in the derby fishery still make more effort enter the fishery as long as profits can be made, and in both fisheries quota constraint must be satisfied. Coupling with the market clearing condition, this yields a system of four non-linear equations with four unknowns:

6) $T_{NEW}\big(D_H(p_M, p_M) + D_L(p_M, p_M)\big) + (1 - T_{NEW})D_L(+\infty, p_M) = Q$
7) $T_{NEW}q(\alpha, E_{NEW}) = \alpha Q$
8) $T_{OLD}q(1 - \alpha, E_{OLD}) = (1 - \alpha)Q$
9) $p_M q(1 - \alpha, E_{OLD}) = E_{OLD}$

which respectively stand for 6) market clearing condition, 7) quota constraint for the quota country, 8) quota constraint for the derby country, and 9) zero profit condition for the derby country. The four endogenous variables are p_M, a single price for both high- and low-quality fish, E_{NEW}, instantaneous effort undertaken by fisherman in the first country to adopt individual quotas, E_{OLD}, instantaneous effort in the country remaining with derby fishing, and T_{OLD}, season length in the latter country.

As long as solution to this system of equations exists, it can be tested whether it constitutes market equilibrium. The necessary condition for it to be a market equilibrium is that production of the adopting country is enough to cover demand for high quality fish, that is $q(\alpha, E_{NEW}) \geq D_H(p_M, p_M)$. If this inequality is not satisfied, then the second case must be considered.

In the second case, the demand for the high quality fish is high enough in comparison to the production of the quota adopting country so that the price of high quality fish outside derby season and inside quota season can be higher than price of the low quality fish. Note that price of low quality fish remains the same throughout the entire year and price of high quality fish can be higher only if fishermen in the quota country market their fish solely as high quality fish. The market equilibrium thus encompasses two prices $p_{LO} < p_{HI}$, the first one for high- and low-quality fish during the derby season and for low-quality fish outside the derby season and the latter for high-quality fish outside the derby season but inside the quota season.

Note that the fishermen in the quota country face two prices when making decision when to undertake their effort. The effort will be chosen, so that the profit margins are equalized within the entire quota season. Therefore, for different prices, efforts will be also different but will be the same within periods where price remains constant. Thus, two levels of effort will be attained in the market equilibrium by the quota fishermen, E_{NEW1} and E_{NEW2} where the former is a level of effort undertaken inside derby season and the latter is the level of effort undertaken outside the derby season. This can be summed up in the following system of equations characterizing market equilibrium:

10) $T_{OLD}\big(D_H(p_{LO}, p_{LO}) + D_L(p_{LO}, p_{LO})\big) + (T_{NEW} - T_{OLD})\big(D_H(p_{HI}, p_{LO}) + D_L(p_{HI}, p_{LO})\big) +$
$(1 - T_{NEW})D_L(+\infty, p_{LO}) = Q$
11) $q(\alpha, E_{NEW2}) = D_H(p_{HI}, p_{LO})$
12) $T_{OLD}q(1 - \alpha, E_{OLD}) = (1 - \alpha)Q$
13) $T_{OLD}q(\alpha, E_{NEW1}) + (T_{NEW} - T_{OLD})q(\alpha, E_{NEW2}) = \alpha Q$
14) $p_{LO}q(1 - \alpha, E_{OLD}) = E_{OLD}$
15) $\frac{p_{LO}q(\alpha, E_{NEW1})}{E_{NEW1}} = \frac{p_{HI}q(\alpha, E_{NEW2})}{E_{NEW2}}$

where the equations respectively stand for 10) market clearing condition for most of the market, 11) market clearing condition for the high quality fringe outside derby season, 12) quota constraint for the derby country, 13) quota constraint for the quota country, 14) zero profit condition for the derby country, 15) equal revenues per unit of effort for the fishermen in the quota country. The endogenous variables are p_{LO}, p_{HI}, T_{OLD}, E_{OLD}, E_{NEW1}, E_{NEW2}.

Similarly to the previous case, as long as a solution exists, it can be checked against a necessary condition $p_{HI} \geq p_{LO}$. If this condition is not satisfied, the solution does not characterize market equilibrium and the first case must be considered. Note that it is possible to obtain $p_{HI} = p_{LO}$ and $q(\alpha, E_{NEW2}) = D_H(p_{HI}, p_{LO})$ as a solution to the above system. In such an event, the previous system of equations should yield the same results with $p_M = p_{HI} = p_{LO}$ and $E_{NEW1} = E_{NEW2} = E_{NEW}$.

3. General results

Lemma 1. Denote $D(p) = D_H(p, p) + D_L(p, p)$. Then $D'(p) < 0$.

Proof. Note that $\lim_{p\to+\infty} D(p) = 0$ and $\forall p \geq 0\ D(p) \geq 0$. Assume now that there exists a positive length interval $A = \left(\underline{p}, \overline{p}\right)$ such that $\forall p \in A\ D'(p) \geq 0$. Since D is not decreasing in this interval and we know that it must eventually decrease to zero as price tends to plus infinity, from the continuity of D, either it is increasing in A and it must have a peak somewhere or it must be constant in A. Either way, there exist $p_1 < p_2$ such that $D(p_1) = D(p_2)$. Let's investigate consumption bundles corresponding to price levels p_1 and p_2. Consumption bundle corresponding to p_1 can be found by solving $\max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L)$ s.t. $q_H + q_L = D(p_1)$ and consumption bundle corresponding to p_2 can be found with $\max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L)$ s.t. $q_H + q_L = D(p_2)$. These in fact are the same optimization problems, and since objective function is strictly concave, the bundles are the same. Denote consumption at this bundle as (q_H^*, q_L^*). Note that $\frac{\partial W}{\partial q_H}(q_H^*, q_L^*) = p_1$ and $\frac{\partial W}{\partial q_H}(q_H^*, q_L^*) = p_2$ must be satisfied at the same time which contradicts $p_1 \neq p_2$ and thus by contradiction A cannot exist. ∎

Lemma 2. $D(p) > D_L(+\infty, p)$ for all $p > 0$.

Proof. It will be sufficient to prove that $\frac{\partial D_H}{\partial p_H} + \frac{\partial D_L}{\partial p_H} < 0$. Note that demands are characterized by the following system of two equations and two unknowns:

$$\begin{cases} \dfrac{\partial W}{\partial q_H}(q_H, q_L) & = & p_H \\ \dfrac{\partial W}{\partial q_L}(q_H, q_L) & = & p_L \end{cases} \Rightarrow \begin{cases} \dfrac{\partial^2 W}{\partial q_H^2} dq_H & + & \dfrac{\partial^2 W}{\partial q_L \partial q_H} dq_L & = & dp_H \\ \dfrac{\partial^2 W}{\partial q_L \partial q_H} dq_H & + & \dfrac{\partial^2 W}{\partial q_L^2} dq_L & = & dp_L \end{cases}$$

Which in turn can be solved for dq_H and dq_L:

$$\begin{cases} dq_H & = & \dfrac{\frac{\partial^2 W}{\partial q_L^2} dp_H}{\frac{\partial^2 W}{\partial q_H^2}\frac{\partial^2 W}{\partial q_L^2} - \left(\frac{\partial^2 W}{\partial q_L \partial q_H}\right)^2} & - & \dfrac{\frac{\partial^2 W}{\partial q_L \partial q_H} dp_L}{\frac{\partial^2 W}{\partial q_H^2}\frac{\partial^2 W}{\partial q_L^2} - \left(\frac{\partial^2 W}{\partial q_L \partial q_H}\right)^2} \\ dq_L & = & \dfrac{\frac{\partial^2 W}{\partial q_H^2} dp_L}{\frac{\partial^2 W}{\partial q_H^2}\frac{\partial^2 W}{\partial q_L^2} - \left(\frac{\partial^2 W}{\partial q_L \partial q_H}\right)^2} & - & \dfrac{\frac{\partial^2 W}{\partial q_L \partial q_H} dp_H}{\frac{\partial^2 W}{\partial q_H^2}\frac{\partial^2 W}{\partial q_L^2} - \left(\frac{\partial^2 W}{\partial q_L \partial q_H}\right)^2} \end{cases}$$

Notice that $\frac{\partial D_H}{\partial p_H} + \frac{\partial D_L}{\partial p_H} = \frac{dq_H}{dp_H} + \frac{dq_L}{dp_H} = \frac{\frac{\partial^2 W}{\partial q_L^2} - \frac{\partial^2 W}{\partial q_L \partial q_H}}{\frac{\partial^2 W}{\partial q_H^2}\frac{\partial^2 W}{\partial q_L^2} - \left(\frac{\partial^2 W}{\partial q_L \partial q_H}\right)^2} = \frac{dq_H}{dp_H} + \frac{dq_H}{dp_L} < 0$ since own price effects are stronger than cross-price effects. ∎

Lemma 3. $\frac{d}{dp} V(p, p) < 0$ and $V(+\infty, p) < V(p, p)$.

Proof. Recall that $V(p, p) = \max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L) - p(q_H + q_L)$ and since W is concave and increasing in both arguments, this optimization problem is equivalent to $\max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L)$ subject to $q_H + q_L \leq D(p)$. Since $D'(p) < 0$ (Lemma 1), increase in p means decreasing the size of the constraint set which in turn means decrease in the objective value as solution is always on the outer boundary.

Note that $V(+\infty, p) = \max_{q_L \geq 0} W(0, q_L) - q_L p \leq \max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L)$ s.t. $q_H + q_L = D(+\infty, p)$

$< \max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L)$ s.t. $q_H + q_L = D(p, p)$

$$= \max_{q_H \geq 0, q_L \geq 0} W(q_H, q_L) - p(q_H + q_L) = V(p, p)$$

■

Existence and uniqueness under individual quotas are straightforward. (2) is a single equation with one unknown, so E_{IQ} can be uniquely determined as long as this equation has a unique solution. Sufficient condition for existence of the solution is that $\lim_{E \to +\infty} q(1, E) > Q/T_{IQ}$ and sufficient condition for uniqueness is that q is strictly increasing. Similarly, (1) is a single equation with one unknown. Notice that in concordance with Lemma 1, left hand side of this equality is strictly decreasing with p_{IQ}. This guarantees uniqueness. For the solution to exist there must be a price yielding demand high enough to satisfy the quota. Finally, even if solution to the system of equation exists, one more condition must be met for this solution to be an equilibrium. Namely, $p_{IQ} q\left(1, E_{IQ}\right) \geq E_{IQ}$. Otherwise, the fishermen will lose money, will have incentives to reduce their catch and will not fulfill the quota. In this case, market equilibrium will be described by different system of equations but this will not be pursued because of its little practical relevance.

Conditions characterizing market equilibrium in derby fishery are more complicated. Let's consider uniqueness first.

Proposition 1. There is a unique market equilibrium in a derby fishing setting.

Proof. Note that two different market equilibria need to have two different prices, since (5) relates uniquely price to effort and (4) relates uniquely effort to season length. So let's assume that there are two market equilibria with two different prices $p_1 < p_2$. According to (5) this implies that corresponding efforts are $E_1 < E_2$ and corresponding season lengths according to (4) are $T_1 > T_2$. The two equations (4) and (5) can be in fact combined to create a strictly decreasing function $T(p)$. We are going to prove that the similar function implicit in equation (3) is strictly increasing which will contradict both p_1 and p_2 being solutions. Let's solve for T using (3):

$$T = \frac{Q - D_L(+\infty, p)}{D_H(p, p) + D_L(p, p) - D_L(+\infty, p)} = \frac{Q - D_L(+\infty, p)}{D(p) - D_L(+\infty, p)}$$

Note that the numerator as well as the denominator is positive, in concordance with Lemma 2. Let's differentiate this expression with respect to p.

$$\frac{dT}{dp} = \frac{-\frac{\partial}{\partial p} D_L(+\infty, p)\big(D(p) - D_L(+\infty, p)\big) - \left(D'(p) - \frac{\partial}{\partial p} D_L(+\infty, p)\right)\big(Q - D_L(+\infty, p)\big)}{\big(D(p) - D_L(+\infty, p)\big)^2}$$

$$\frac{dT}{dp} = \frac{\frac{\partial}{\partial p} D_L(+\infty, p)\big(Q - D(p)\big) - D'(p)\big(Q - D_L(+\infty, p)\big)}{\big(D(p) - D_L(+\infty, p)\big)^2} > 0$$

The above expression is greater than zero since derivatives of both demand functions are negative. Moreover, since quota is a weighted average of the two demands, and $D(p) > D_L(+\infty, p)$, then $Q - D(p) < 0$ and $Q - D_L(+\infty, p) > 0$. ■

Equation (5) has at least one solution $E = 0$ given price. This is not the solution we are interested in, since it has little practical relevance (and is unstable when another solution exists). Positive solution exists when the price is high enough to justify extraction at a rate allowing for meeting the quota within one year. Otherwise, quota will not be met but this outcome will not be pursued because of its little practical significance. Moreover, if q function has a limit as E tends to plus infinity, there may exists no level of effort allowing to satisfy the quota (equation (4)). Therefore, the solution to this system may not exist under some circumstances, but these cases are not likely to happen.

Let's now consider comparative statics for derby fishing. The results are presented in the form of Proposition 2. Note that at least one result is not intuitive: as we increase quota, instantaneous effort decreases. The reason for that is that price decreases as well, so fishermen have to reduce their effort to break even at each instant of time.

Proposition 2. In a derby fishery equilibrium $\frac{dp}{dQ} < 0, \frac{dE}{dQ} < 0, \frac{dT}{dQ} > 0, \frac{dCS}{dQ} > 0$.

Proof. We can totally differentiate the system

$$\begin{cases} q dp & + & (pq' - 1) dE & + & 0 dT & = & 0 \\ 0 dp & + & Tq' dE & + & q dT & = & dQ \\ \left[TD'(p) + (1 - T)\frac{\partial}{\partial p} D_L(+\infty, p)\right] dp & + & 0 dE & + & [D(p) - D_L(+\infty, p)] dT & = & dQ \end{cases}$$

Let's solve for implicit derivatives using Cramer's rule. Note that

$$W = \begin{vmatrix} q & (pq' - 1) & 0 \\ 0 & Tq' & q \\ TD'(p) + (1 - T)\frac{\partial}{\partial p} D_L(+\infty, p) & 0 & D(p) - D_L(+\infty, p) \end{vmatrix} = \begin{vmatrix} q & b & 0 \\ 0 & c & q \\ e & 0 & f \end{vmatrix} = q(cf + be) > 0$$

since $q > 0$, $c > 0$, $f > 0$ (see Lemma 2), $b < 0$ (from (5) and concavity of q), and $e < 0$ (as weighted average of two negative numbers). Then

$$W_p = \begin{vmatrix} 0 & b & 0 \\ dQ & c & q \\ dQ & 0 & f \end{vmatrix} = b(q - f)dQ, W_E = \begin{vmatrix} q & 0 & 0 \\ 0 & dQ & q \\ e & dQ & f \end{vmatrix} = q(f - q)dQ, W_T = \begin{vmatrix} q & b & 0 \\ 0 & c & dQ \\ e & 0 & dQ \end{vmatrix} = (qc + be)dQ$$

Additionally $q - f = q - D(p) + D_L(+\infty, p_{DF}) > 0$ hence:

$$\frac{dp}{dQ} = \frac{W_p}{W} = \frac{b(q-f)}{q(cf+be)} < 0, \frac{dE}{dQ} = \frac{W_E}{W} = \frac{f-q}{cf+be} < 0, \frac{dT}{dQ} = \frac{qc+be}{q(cf+be)} > 0$$

Now, let's focus on $\frac{dCS}{dQ}$ and $\frac{d(TE)}{dQ}$.

$$\frac{dCS}{dQ} = \frac{d}{dQ}[TV(p,p) + (1-T)V(+\infty,p)] > 0$$

since $\frac{dT}{dQ} > 0$, $\frac{dp}{dQ} < 0$ and $\frac{d}{dp}V(p,p) < 0$, $\frac{d}{dp}V(+\infty,p) < 0$, and $V(+\infty,p) < V(p,p)$ (Lemma 3). Consumer surplus is a weighted average and by increasing total quota we increase both components of this average and put more weight on the bigger component. ■

Proposition 3. In the individual quota fishery $\frac{dE}{dT} < 0, \frac{dE}{dQ} > 0, \frac{dp}{dT} > 0, \frac{dp}{dQ} < 0, \frac{d\pi}{dT} > 0, \frac{d\pi}{dQ} \gtreqless 0, \frac{dCS}{dT} \lesseqgtr 0,$ $\frac{dCS}{dQ} > 0$, where π is the total industry profit.

Proof. Let's totally differentiate (2):

$$q(1,E)dT + Tq'(1,E)dE = dQ$$

Therefore $\frac{dE}{dT} = -\frac{q(1,E)}{Tq'(1,E)} < 0$ and $\frac{dE}{dQ} = \frac{1}{Tq'(1,E)} > 0$. Now, let's totally differentiate (1):

$$TD'(p)dp + D(p)dT + (1-T)\frac{\partial}{\partial p}D_L(+\infty,p)dp - D_L(+\infty,p)dT = dQ$$

Hence we have $\frac{dp}{dT} = -\frac{D(p)-D_L(+\infty,p)}{TD'(p)+(1-T)\frac{\partial}{\partial p}D_L(+\infty,p)} > 0$ and $\frac{dp}{dQ} = \frac{1}{TD'(p)+(1-T)\frac{\partial}{\partial p}D_L(+\infty,p)} < 0$.

The profit is defined as $\pi = Qp - ET$. Let's totally differentiate: $d\pi = Qdp + pdQ - EdT - TdE$. Now $\frac{d\pi}{dT} = Q\frac{dp}{dT} - E - T\frac{dE}{dT}$. The last two terms could be aggregated to $E + T\frac{dE}{dT} = E - \frac{q(1,E)}{q'(1,E)} < 0 \Leftrightarrow$ $\frac{q(1,E)}{E} > q'(1,E)$ and the latter inequality is always true since q is strictly concave and $q(s,0) = 0$. Note also that $\frac{dp}{dT} > 0$ therefore $\frac{d\pi}{dT} > 0$ as it is a sum of two positive expressions.

$\frac{d\pi}{dQ} = Q\frac{dp}{dQ} + p - T\frac{dE}{dQ}$ where the first two terms can be interpreted as elasticity effect and their sum can be either positive or negative. The last term can be interpreted as effort effect and is always negative. Thus the sign of the whole expression is ambiguous.

Consumer surplus is represented by $CS = TV(p,p) + (1-T)V(+\infty,p)$. When we totally differentiate we get $dCS = T\frac{d}{dp}V(p,p)dp + V(p,p)dT + (1-T)\frac{d}{dp}V(+\infty,p)dp - V(+\infty,p)dT$. In turn, we get:

$$\frac{dCS}{dT} = V(p,p) - V(+\infty,p) + \left(T\frac{d}{dp}V(p,p) + (1-T)\frac{d}{dp}V(+\infty,p)\right)\frac{dp}{dT}$$

Note that first two terms are always positive (Lemma 3) and the last term is always negative. Therefore the effect of expanding season length on consumer surplus is ambiguous. On the other hand

$$\frac{dCS}{dQ} = \left[T\frac{d}{dp}V(p,p) + (1-T)\frac{d}{dp}V(+\infty,p)\right]\frac{dp}{dQ} > 0$$

since both multiplicands are less than zero. ■

Note that as we increase the length of the fishing season, the price increases as well. This comes from the fact that increase in fishing season allows more fish to be marketed as fresh and since the price does not change within the year, the overall price should be higher. Note also that price decreases with total quota which allows us to derive the annual demand function. This demand function is different under different regulatory regimes.

Increasing quota will initially increase profits but as quota increases even further, elasticity effect becomes weaker and is eventually dominated by effort effect. At this point, profits become negatively related to quota. Increasing quota even further makes both elasticity effect and effort effect negative. Similarly, it is surprising that effect of prolonging season on consumer surplus is ambiguous. This is because positive effect of having more fresh fish can be countervailed by negative effect of price increases.

4. Numerical experiments

In order to proceed with numerical experiments, a C++ application has been written. The program is capable of computing equilibria in all three cases (derby fishing, individual quotas, and mixed setting) for a given welfare function W, yield-effort function q, and exogenous variables like α, T_{IQ}, etc. It consists of three layers. The first layer is responsible for numerical optimization of the welfare function and computing demands given prices. The second layer implements an algorithm for solving systems of equations and thus finds a market equilibrium in each of the three cases (derby fishing, individual quotas, and mixed). It calls the first layer whenever demands must be evaluated. The third layer consists of a loop which allows for doing comparative static. It iteratively solves for equilibria and changes parameters. The second layer is called upon each iteration of the third layer. The results are stored in a CSV file and can be further analyzed with Excel or a statistical package.

For a numerical simulation to succeed, functions in the model need to take particular forms. A quadratic welfare function of $W(q_H, q_L) = aq_H + bq_L - cq_H^2 - dq_L^2 - eq_Hq_L$ was used to approximate the true welfare function. Note that this function is strictly concave for $4cd - e^2 > 0$ and is increasing in the relevant rage that is for quantities corresponding to positive prices. This functional form is also interesting because it allows for use of analytical demands rather than numerical demands, which improves speed and numerical precision of the simulation. A functional form of the instantaneous production function was assumed to be $q(s,E) = As^{1-\rho}E^{\rho}$.

The model has been calibrated to resemble the historical situation in North Pacific halibut fishery from the early 1990s. Five data points were used to determine unknown parameters of the welfare function. It was assumed that before IVQs were introduced in British Columbia, of the total quota of 55 million pounds, 3.85 million pounds were sold fresh and 51.15 million pounds were sold frozen. The average ex vessel price was assumed to be $1.70 per pound and the season length was 0.03 which corresponds to 11 days. Moreover, during the period between 1991 and 1995 when British Columbia enjoyed quotas while Alaska was still in derby fishing, the amount of fish sold by Canadian fishermen as high quality fish was assumed to be 7.05 million pounds while total quota was 45 million pounds. Price for that fish was $2.40 and the price for the rest of the fish was $1.90. Finally, after both countries ended up with quota systems, price of fish stabilized at $2.10 with the season length of 0.68 (248 days) and total quota of 60 million pounds. It was assumed that $\alpha = \frac{1}{6}$. The five data points are summarized in Table 1.

Period	Interpretation	Equation
Pre-IVQ	Total fresh sales.	$0.03D_H(1.7{,}1.7) = 3.85$
Pre-IVQ	Total frozen sales.	$0.03D_L(1.7{,}1.7) + 0.97D_L(+\infty, 1.7) = 51.15$
Mixed	Fresh sales outside Alaskan derby season.	$(0.68 - 0.03)D_H(2.4{,}1.9) = 7.05$
Mixed	Remaining sales.	$0.03\big(D_H(1.9{,}1.9) + D_L(1.9{,}1.9)\big) + (0.68 - 0.03)D_L(2.4{,}1.9) + 0.32D_L(+\infty, 1.9) = 37.95$
Post-ITQ	Total sales.	$0.68\big(D_H(2.1{,}2.1) + DL(2.1{,}2.1)\big) + 0.32D_L(+\infty, 2.1) = 60$

Table 1. Equations and their interpretation allowing for calibration of the demand side of the model.

Assuming quadratic welfare function, the above system of equations turns out to be a system of five non-linear equations with five unknowns. It was subsequently solved with Excel and MATLAB. The results are shown in Table 2.

```
a = 2.62520771509542;
b = 2.43120953312058;
c = 0.003397420093471;
d = 0.006983084613824;
e = 0.004374020032835;
```

Table 2. Results of the calibration of the demand side of the model.

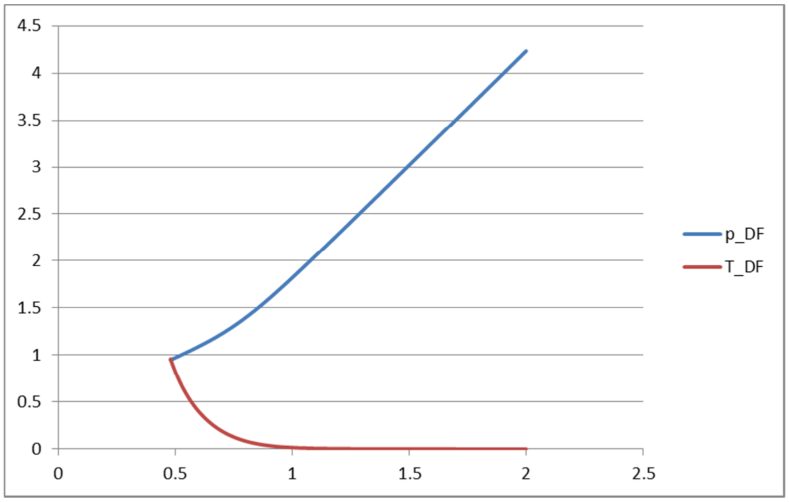
The parameters of the instantaneous production function were determined based on situation in pre-IVQ and post-ITQ periods. During the derby period, total revenues and the season length were used to determine level of instantaneous effort: $E_{DF} = \frac{p_{DF}Q_{DF}}{T_{DF}} = 1.7 \times \frac{55}{0.03} = 3116\frac{2}{6}$ (interpretation: over 3 bln USD/year). During the individual quotas period, it was assumed that profits constitute 50% of revenues, based on the historical quota leasing prices (Pinkerton and Edwards, 2009). Therefore, the instantaneous effort was determined as $E_{IQ} = \frac{p_{IQ}Q_{IQ}}{2T_{IQ}} = \frac{2.1\times 60}{2\times 0.68} \approx 92.6471$. These were then used to determine two parameters of the instantaneous production function by solving the system of two non-linear equations with two unknowns with Excel and MATLAB.

$$\begin{cases} q(1, E_{DF}) & = & \dfrac{Q_{DF}}{T_{DF}} \\ q\big(1, E_{IQ}\big) & = & \dfrac{Q_{FQ}}{T_{FQ}} \end{cases} \Rightarrow \begin{cases} A\left(3116\dfrac{2}{6}\right)^{\rho} & = & 1833\dfrac{1}{3} \\ A(92.65)^{\rho} & = & 88.2353 \end{cases} \Rightarrow \begin{cases} A & = & 1.77161626546777 \\ \rho & = & 0.862947619417066 \end{cases}$$

The parameters of the model have been found so the next step would be to test model's predictions against an additional data point. We decided to use a ratio of Canadian production rate outside Alaskan derby season to Canadian production rate inside Alaskan derby season during the mixed regulation period. We used Figure 3 from Casey et al. (1995) as a rough indicator of what this ratio was in practice. Since our model yields equilibrium effort levels of 1.98 and 11.25 for the inside and outside of the Alaskan derby season respectively, the corresponding instant catches, given above values of A and ρ are 3.20 and 14.31 million pounds a year. And their ratio is 0.22. This indeed roughly corresponds to what we observe on Figure 3 in Casey et al., when we compare the average catch in March, April, May, July and October to average catch in June, August and September. Note that we do not compare absolute values and we focus on the ratio instead. This is because the aforementioned Figure 3 was created only for year 1993 which had higher total quota than 45 million pounds assumed by us as an average for the mixed regulation period.

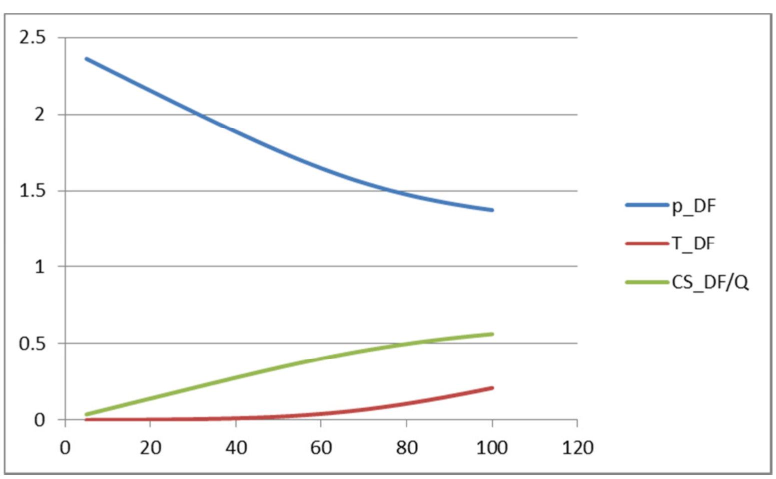
Once the model has been parameterized, it was possible to do all sort of simulations. Please note that the figures above were chosen to roughly reflect historical data and were not estimated precisely. The main purpose of the numerical experiment is to illustrate features of the model, rather than give precise predictions.

In the first simulation we wanted to test the model against the results obtained by Homans and Wilen (2005). In their paper, the authors show how increase in demand can affect the length of the season under derby fishing. For some reasons, authors disregard effects of joint increases in demands and show separately effects of an increase in the demand for high quality fish and effects of an increase in low quality fish. This would be hard to do in our model, because demands for low- and high-quality fish are determined jointly by a single welfare function. Nevertheless, the comparison of the results is interesting. Picture 1 shows how price and season length changes with increasing demand. The increase in demand in this simulation was modeled by multiplying intercepts of the demand functions by the *demand multiplier*. In our case the intercepts are a and b parameters of the welfare function for the high- and low-quality products respectively. Note that demand multiplier equal to one yields season length of 0.03 but decreasing demand level by 30% results in the season length of 0.19 which corresponds to almost 70 days. The season length increases dramatically as we proceed further with the decrease in demand. When demand drops by slightly more than a half (demand multiplier < 0.48) the potential season length exceeds one year, and equilibrium given by the system of equations (3)-(5) cannot be reached – the fishery is no longer able to meet the quota constraint and delivers less than total allowable catch. This dramatic change is consistent in direction and inconsistent in magnitude with the findings of Homans and Wilen (2005). Their simulation does not allow answering why season length can decrease to such a small fraction of a year, in fact the results of their simulations yield season lengths of more than a month (more than 0.14). Even for increases in demand scaling parameter (which corresponds to our demand multiplier) of the magnitude of 10, the change in their model is not so dramatic as with the demand rising by several percent in our model. As it is likely that over several years demand increased by several percent in the North Pacific Halibut Fishery, rather than more than tenfold, we find our model more consistent with the empirical evidence.



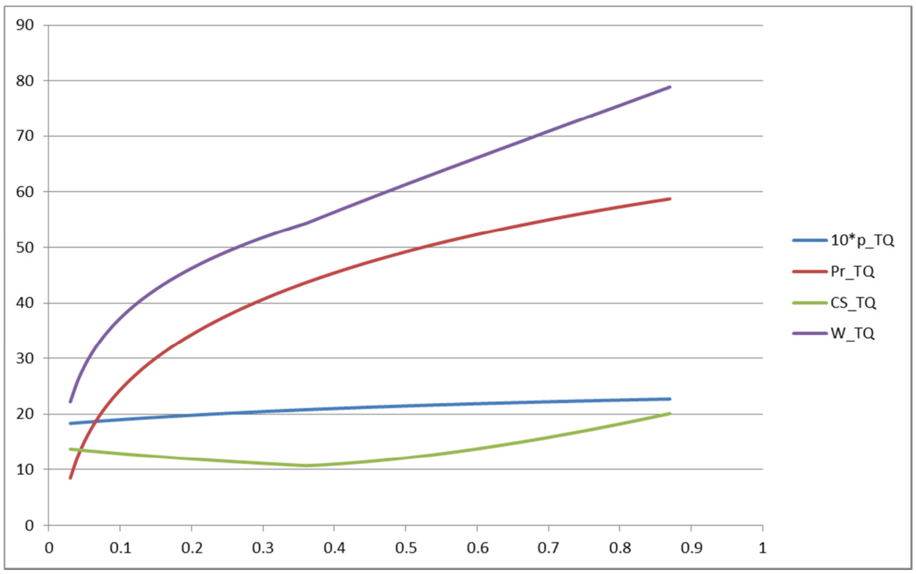
Picture 1. Relation between the demand multiplier and price / season length under derby fishing.

It is also interesting to see what are the effects of increasing quota in a derby fishery. Picture 2 shows that price (blue line), season length (red line) as well as consumer surplus per unit of output (green line) increase with the quota. We decided to use consumer surplus per unit of output since it was obvious that consumer surplus would increase as both price is decreasing in season length is increasing. It is notable, that consumer surplus per unit of output is increasing but at a decreasing rate. The other two endogenous variables change as predicted by our comparative statics. We can see here again, that as long as quota gets high comparable to the demand level, the season length increases considerably.



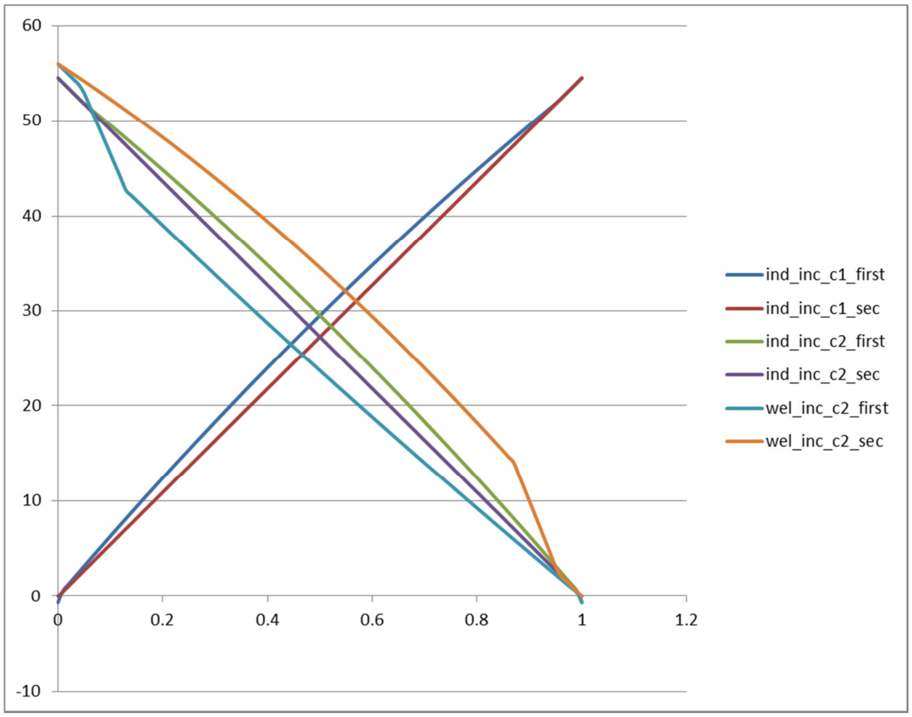
Picture 2. Relationship between prices, season length, and consumer welfare per unit of output and quota (in million pounds).

Our next simulation will present how season length under individual quota program affects prices, profits, consumer surplus and total welfare. The results are presented in Picture 3. On the horizontal axis we have T_{IQ}. Blue line represents price multiplied by 10, red line represents profits, green line represents consumer surplus and the violet line represents total welfare. Note that consistently with the results of comparative statics obtained in the previous chapter, price and profits are increasing with season length in the individual quota fishery. Moreover, we can see that also consistently with the predictions of comparative static, consumer surplus is decreasing initially and then rebounds. Total welfare increases with season length.



Picture 3. Relationship between prices, profits, consumer surplus, and total welfare and length of the season under individual quotas.

Finally, we would like to focus on incentives each country faces while adopting the individual quota regulation. These are depicted on Picture 4. Note that gains in industry profit from being the first country to adopt the efficient regulation are always greater than from being the second country to adopt the efficient regulation but the difference is small. Total welfare gains are considerably higher when the country is the second one to adopt individual quotas. Note that in our example only one country is considered to be recipient of consumer surplus. The reason for that is that in North Pacific Halibut Fishery most of the product from both Alaska and British Columbia is subsequently sold in the US market. This may at least partially explain why Alaska was so eager to adopt the innovative regulation soon after it was done in British Columbia.



Picture 4. "ind_inc" stands for industry incentives, that is incentives based on profit only. "wel_inc" are incentives based on both profits and consumer surplus. c1 is country one, that is the country whose size is measured by α. c2 is country two, whose size is $1-\alpha$. "first" or "sec" indicates whether this country is first or second to adopt the individual quota regulation. Incentives, as changes in profits and welfare, are in millions of dollars. The variable on horizontal axis is α.

5. Conclusions

We proposed the first intra-seasonal model of a fishery allowing modeling both efficiency gains and welfare effects of introducing individual quotas. Moreover, the general framework we are using is suitable for modeling regulatory designs other than individual quotas or derby fishing, for example when a fishery is divided between two areas regulated in different ways. The framework we developed here is static, in the form of system of equations with unique solutions, and as such allows for easy derivation of comparative statics. If this is not enough, after assuming particular functional forms, it is possible to calibrate the model to resemble particular fishery and then change parameters to obtain counterfactual outcomes.

We compare assumptions and findings of our model to results obtained by Homans and Wilen (2005). We find that some of their assumptions are unnecessary and some are counterintuitive or even contradictory to empirical findings. Results of our model are also more consistent with what happened in the real example fishery. Finally, we are able to perform welfare analysis and model efficiency gains from implementing individual quotas, both of which seem to be hard to do with the model derived by Homans and Wilen.

Comparative statics we derived as well as simulations of North Pacific Halibut Fishery allow us to proceed with some valuable policy recommendation. Probably the most important policy recommendation is to increase the length of the individual quota season. This always have a positive effect on profits of the fishermen and increases consumer surplus, as long as season length is a considerable fraction of the year to begin with (more than 6 months for example). This argument though does not take into account biological arguments like protection of the species during the migration period.

Another policy implication is for international fisheries. If there are two countries sharing a fishery and one of the countries is a primary destination of the product, then it may be easier to convince the other country to adopt the individual quota regulation first, as its industry gains from adopting it first are greater than industry gains from adopting it as a second country (the overall gains when both countries eventually adopted individual quotas are independent of the order of adoption). On the other hand, the country which constitutes market for the product, faces significant surplus loss induced by the spike in price which lowers the overall benefits of adopting individual quotas as a first country. This surplus loss can also by induced by the other country adopting individual quotas first, which lays ground for much higher gain for adoption as a second country.

Finally, the research presented here is certainly not complete and many further topic emerge as extensions or improvements to this study. First of all, it may be possible to derive comparative statics for the mixed model, where one of the countries adopted the new regulatory regime meanwhile the other has not done so yet. We abstained from doing that in this paper. Secondly, numerous other papers develop quantitative models and make predictions (see for example Herrmann and Criddle, 2006). The results obtained by them could be easily compared with the results of simulations that can be done with our model. It may also be interesting to try to relax some assumption in our model by for example assuming existence of processing sector. Finally, our model does not explain why Alaskan price dropped below its pre IVQs price during the mixed period. We leave that for future research.

Literature

Casey K., C. Dewees, B. Turris, J. Wilen, The effects of individual vessel quotas in the British Columbia Halibut Fishery, Marine Resource Economics, 1995.

Christy F., Fishermen quotas: a tentative suggestion for domestic management, Occasional paper 19, Law of the Sea Institute, University of Rhode Island, Kingston, 1973.

Clark C., Mathematical bioeconomics: the optimal management of renewable resource, John Wiley & Sons, 1976.

Clark C., Towards a predictive model for the economic regulation of commercial fisheries, Canadian Journal of Fisheries and Aquatic Sciences, Vol. 37, 1980.

Clark C., The worldwide crisis in fisheries, economic models and human behavior, Cambridge University Press, 2006.

Copes P., The backward-bending supply curve of the fishing industry, Scottish Journal of Political Economy, v. 17, 1970.

Gordon S., The economic theory of a common-property resource: the fishery, Journal of Political Economy, Vol. 62, 1954.

Herrmann M., Estimating the induced price increase for Canadian pacific halibut with the introduction of the individual vessel quota program, Canadian Journal of Agricultural Economics 44, 1996.

Herrmann M., K. Criddle, An econometric market model for Pacific Halibut Fishery, Marine Resource Economics, Vol. 21, 2006.

Homans F., J. Wilen, Markets and rent dissipation in regulated open access fisheries, Journal of Environmental Economics and Management 49, 2005.

Matulich S., R. Mittelhammer, C. Reberte, Towards a more complete model of individual transferable fishing quotas: implications of incorporating the processing sector, Journal of Environmental Economics and Management 31, 1996.

Newell R., J. Sanchirico, S. Kerr, Fishing quota markets, Journal of environmental economics and management 49, 2005.

Pinkerton E., D. Edwards, The elephant in the room: the hidden costs of leasing individual transferable fishing quotas, Marine Policy 33, 2009.

Weninger Q., Assessing efficiency gains from individual trasferable quotas: an application to the mid-Atlantic surf clam and ocean quahog fishery, American Journal of Agricultural Economics, Vol. 80, 1998.